(P3) (DIFFERENTIATION)

Base: Log, Trig $\longrightarrow$ DIFF, INTEG, DIFF. EQ

OPERATORS

[1] POWER $\quad (\square)^n \longrightarrow n(\square)^{n-1} \times \square'$

[2] TRIG

$$\sin\square \longrightarrow \cos\square \times \square'$$

$$\cos\square \longrightarrow -\sin\square \times \square'$$

$$\tan\square \longrightarrow \sec^2\square \times \square'$$

[3] EXP $\quad e^{\square} \longrightarrow e^{\square} \times \square'$

[4] LOGS $\quad \ln\square \longrightarrow \dfrac{1}{\square} \times \square'$

[5] Inverse Tan: $\quad \tan^{-1}\square \longrightarrow \dfrac{1}{1+\square^2} \times \square'$

NOTATION :

$$y \xrightarrow{\text{Diff}} \frac{dy}{dx} \xrightarrow{\text{diff}} \frac{d^2y}{dx^2}$$

$$y \longrightarrow y' \longrightarrow y''$$

$$f(x) \longrightarrow f'(x) \longrightarrow f''(x)$$

SINGLE OPERATORS

1) $y = (2x-3)^6$

$y = (\boxed{2x-3})^{\underline{\underline{6}}}$

$\frac{dy}{dx} = 6(2x-3)^5(2) = 12(2x-3)^5$

2) $Y = \sin(2x)$

$Y = \underline{\underline{\sin}}(\boxed{2x})$

$\frac{dy}{dx} = \cos(2x) \times 2$

$\frac{dy}{dx} = 2\cos 2x$

3) $y = 2\,\underline{\underline{\cos}}\left(\boxed{2x + \frac{\pi}{6}}\right)$

$\frac{dy}{dx} = 2\left(-\sin\left(2x + \frac{\pi}{6}\right)\right) \times (2)$

$\frac{dy}{dx} = -4\sin\left(2x + \frac{\pi}{6}\right)$

3) $y = \tan 3x$

$y = \tan \boxed{3x}$

$\frac{dy}{dx} = \sec^2 3x \times 3 = 3\sec^2 3x$

4) $y = e^{2x+1}$

$y = e^{\boxed{2x+1}}$

$\frac{dy}{dx} = e^{2x+1} \times 2 = 2e^{2x+1}$

5) $y = \ln(2x+5)$

$y = \ln(\boxed{2x+5})$

$\frac{dy}{dx} = \frac{1}{2x+5} \times 2 = \frac{2}{2x+5}$

6) $y = \tan^{-1}(3x)$ $\qquad \tan^{-1}\square \longrightarrow \frac{1}{1+\square^2} \times \square'$

$y = \tan^{-1}(\boxed{3x})$

$$\frac{dy}{dx} = \frac{1}{1+\boxed{3x}^2} \times \boxed{3}'$$

$$= \frac{1}{1+9x^2} \times 3$$

$$\boxed{\frac{dy}{dx} = \frac{3}{1+9x^2}}$$

MULTIPLE OPERATORS

OUTSIDE OPERATOR IS DIFF FIRST.

INSIDE OPERATOR = WHICH LIES INSIDE BOX OF ANOTHER OPERATOR

$$y = \sin^3 x$$

$$y = (\boxed{\sin \boxed{x}})^3$$

power = outside operator

sin = inside operator.

$$\frac{dy}{dx} = 3(\sin x)^2 \times (\cos x \times 1)$$

$$\frac{dy}{dx} = 3\sin^2 x \cos x$$

(ii) $y = e^{\boxed{\tan \boxed{3x}}}$

e = outside

tan = inside

$\tan\square \longrightarrow \sec^2\square \times \square'$

$$\frac{dy}{dx} = e^{\tan 3x} \times [\sec^2 3x \times 3]$$

(iii) $y = \ln|\boxed{\cos \boxed{2x}}|$

$\ln \square \longrightarrow \frac{1}{\square} \times \square'$

$$\frac{dy}{dx} = \frac{1}{\cos 2x} \times [-\sin 2x \times 2]$$

(iv) $y = \cos^3 4x$

$$y = (\boxed{\cos \boxed{4x}})^3$$

$$\frac{dy}{dx} = 3(\cos 4x)^2[-\sin 4x \times 4]$$

$$\frac{dy}{dx} = -12\cos^2 4x \sin 4x$$

RULE: Use Product & Quotient rule only when two VARIABLE terms MULTIPLY / DIVIDE.

PRODUCT RULE	QUOTIENT RULE
$(uv)' = uv' + vu'$	$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$
$y = \underbrace{x}_{u}\,\underbrace{\sin 3x}_{v}$	$y = \frac{2x+1}{5x+4}\ \frac{u}{v}$
$\frac{dy}{dx} = x[\cos 3x \times 3] + \sin 3x[1]$	$\frac{dy}{dx} = \frac{(5x+4)[2]-(2x+1)[5]}{(5x+4)^2}$

$\frac{dy}{dx} = 3x\cos 3x + \sin 3x$ | $\frac{dy}{dx} = \frac{10x+8-10x-5}{(5x+4)^2} = \frac{3}{(5x+4)^2}$

$y = \frac{1}{\sin^2 x} = (\sin x)^{-2}$

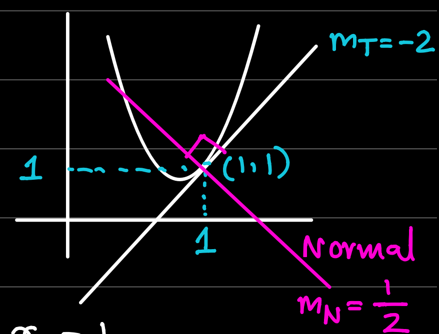
Its not good idea to use quotient rule.

$\frac{dy}{dx} = -2(\sin x)^{-3} \times [\cos x \times 1]$

$\frac{dy}{dx} = -\frac{2\cos x}{\sin^3 x}$.

BASIC OUTCOMES

$y = 2x^2 - 6x + 5$



(i) Find $\frac{dy}{dx} = 4x - 6$

(ii) Find gradient of tangent at $x = 1$

$m_T = 4(1) - 6 = -2$

(iii) Find equation of tangent at $x = 1$

$y = 2x^2 - 6x + 5$

$y = 2(1)^2 - 6(1) + 5 = 1$

$m_T = -2$, $(1, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -2(x - 1)$

$y = -2x + 2 + 1$

TANGENT: $\boxed{y = -2x + 3}$

(iv) Find equation of Normal at $x = 1$

$m_T = -2 \longrightarrow m_N = \frac{1}{2}$ use same point $(1, 1)$

$y - 1 = \frac{1}{2}(x - 1)$

$2y - 2 = x - 1$

NORMAL $2y = x + 1$

(v) Find x coordinate of stationary point.

$\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 4x - 6$

$0 = 4x - 6$

$x = 1.5$

(vi) Find nature of stationary point.

Method 1:

$y = 2x^2 - 6x + 5$

$\frac{dy}{dx} = 4x - 6$

$\frac{d^2y}{dx^2} = 4$ (+ve)

(Min).

Just for practice Lets see method 2.

$$\frac{dy}{dx} = 4x - 6$$

STATIONARY $\boxed{x = 1.5}$

0.1 smaller

0.1 bigger.

$x = 1.4$

$x = 1.6$

$$\frac{dy}{dx} = 4(1.4) - 6$$

$$= -0.4$$

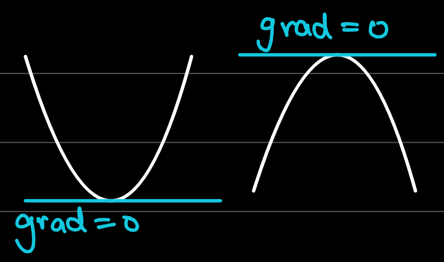
$$\frac{dy}{dx} = 4(1.6) - 6$$

$$0.4$$

$-$ $\longrightarrow$ $+$ (Min).

STATIONARY POINTS:

Turning points
Max. Points
Min. Points
Critical Points
Vertex.



$$\boxed{\frac{dy}{dx} = 0}$$

NATURE OF A STATIONARY POINT

Method 1 is used for P1.
Both Method 1 and method 2 is used for P3.

Method 1

$$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2}$$

$\rightarrow$ +ve (min)
$\rightarrow$ −ve (max)

Method 2 (P3 only): When double derivative is not feasible. When you have to use product or quotient rule for $\frac{d^2y}{dx^2}$

Now you only have $\frac{dy}{dx}$ and x coordinate of stationary point.

STATIONARY $x = \square$

0.1 smaller ← → 0.1 bigger

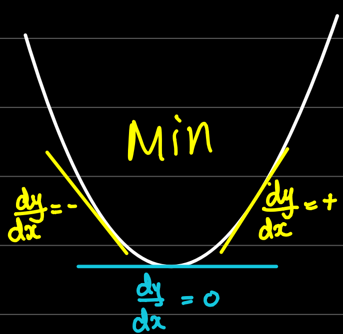
$x = \boxed{}$ $\qquad\qquad$ $x = \boxed{}$

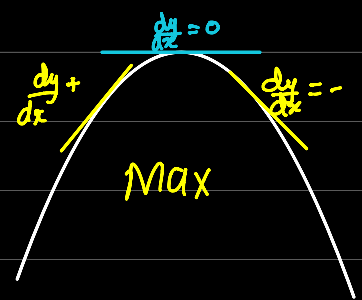
Find $\frac{dy}{dx}$ on both values.

$\frac{dy}{dx} = \boxed{}$ $\qquad\qquad$ $\frac{dy}{dx} = \boxed{}$

If $\quad$ $-$ ⟶ $+$ $\quad$ Min

$\quad\quad$ $+$ ⟶ $-$ $\quad$ Max.





DIFFERENTIATION
INTEGRATION
ITERATION
DIFF. EQUATIONS

} RADIAN MODE.

1 The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

$y = x + \cos\boxed{2x}$ $\qquad 0 \leq x \leq \pi$

$\frac{dy}{dx} = 1 + (-\sin 2x)(2)$

$\frac{dy}{dx} = 1 - 2\sin 2x$

$0 = 1 - 2\sin 2x$

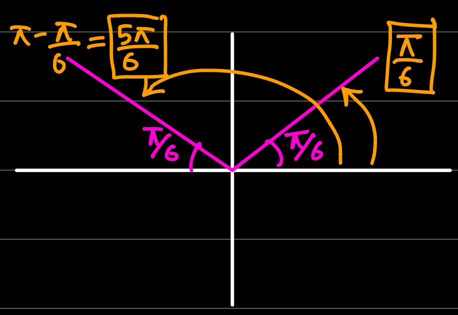
$\sin 2x = \frac{1}{2}$ $\qquad 0 \leq x \leq \pi$

$\times 2$

$2x = A$ $\qquad 0 \leq 2x \leq 2\pi$

$\sin A = \frac{1}{2}$ $\qquad 0 \leq A \leq 2\pi$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



$2x = A = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}}$

$\frac{dy}{dx} = 1 - 2\sin\boxed{2x}$

$$\frac{d^2y}{dx^2} = 0 - 2(\cos 2x)(2)$$

$$\frac{d^2y}{dx^2} = -4\cos 2x$$

$x = \frac{\pi}{12}$, $\frac{d^2y}{dx^2} = -4\cos\left[2\times\frac{\pi}{12}\right] = -2\sqrt{3}$ (max)

$x = \frac{5\pi}{12}$, $\frac{d^2y}{dx^2} = -4\cos\left[2\times\frac{5\pi}{12}\right] = 2\sqrt{3}$ (min)

2 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

(i) $y = 6e^x - e^{3x}$

$$\frac{dy}{dx} = 6e^x(1) - e^{3x}(3)$$

$$\frac{dy}{dx} = 6e^x - 3e^{3x}$$

$$0 = 6e^x - 3e^{3x}$$

$$3e^{3x} = 6e^x$$

$$\frac{e^{3x}}{e^x} = \frac{6}{3}$$

$$\ln e^{2x} = \ln 2$$

$$2x \ln e = \ln 2$$

$$2x(1) = \ln 2$$

$$x = \frac{\ln 2}{2} = 0.347$$

(ii) $\frac{dy}{dx} = 6e^x - 3e^{3x}$

$$\frac{d^2y}{dx^2} = 6e^x(1) - 3e^{3x}(3)$$

$$\frac{d^2y}{dx^2} = 6e^x - 9e^{3x}$$

$x = 0.347$

$$\frac{d^2y}{dx^2} = 6e^{0.347} - 9e^{3(0.347)}$$

$$\frac{d^2y}{dx^2} = -16.995$$

Max.

3 The curve with equation $y = e^{-x} \sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

$$y = \underbrace{e^{-x}}_{u} \underbrace{\sin x}_{v}$$

$$\frac{dy}{dx} = e^{-x}[\cos x \times 1] + \sin x[e^{-x} \times -1]$$

$$\frac{dy}{dx} = e^{-x}(\cos x - \sin x)$$

$$0 = e^{-x}(\cos x - \sin x)$$

$\ln e^{-x} = \ln 0$

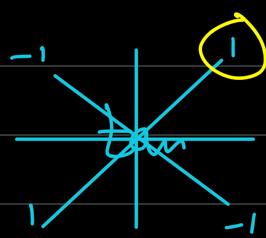
No solution

$\cos x - \sin x = 0$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$



$$\boxed{x = \frac{\pi}{4}}$$

$$\boxed{x = 0.785}$$

(ii) $$\frac{dy}{dx} = e^{-x}(\cos x - \sin x)$$

Here we use method 2.

$$\boxed{x = 0.785}$$

$x = 0.7$ $\qquad\qquad$ $x = 0.8$

$$\frac{dy}{dx} = e^{-0.7}(\cos 0.7 - \sin 0.7) \qquad \frac{dy}{dx} = e^{-0.8}(\cos 0.8 - \sin 0.8)$$

$$\frac{dy}{dx} = 0.0599 \qquad \frac{dy}{dx} = -9.27 \times 10^{-3}$$

$$+ \longrightarrow -$$

Max.

TYPE 2 PARAMETRIC EQUATIONS

You will have separate equations for x & y.
Third variable is called parameter.

Q Find $\frac{dy}{dx}$

$$x = 3 - 2\cos\boxed{2\theta} \qquad y = 5\sin\boxed{2\theta} - 3$$

$$\frac{dx}{d\theta} = 0 - 2[-\sin 2\theta \times 2] \qquad \frac{dy}{d\theta} = 5\cos 2\theta \times 2 - 0$$

$$\frac{dx}{d\theta} = 4\sin 2\theta \qquad \frac{dy}{d\theta} = 10\cos 2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$\frac{dy}{dx} = \frac{10\cos 2\theta}{4\sin 2\theta} = \frac{5}{2}\cot 2\theta$$

9709/33/O/N/11

8 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2\sin\theta.$$

Show that $\frac{dy}{dx} = \frac{2\cos\theta}{1 + 2\sin\theta}$. [5]

9709/33/M/J/12

DIFF

$$x = \sin 2\theta - \theta \qquad y = \cos 2\theta + 2\sin\theta$$

$$\frac{dx}{d\theta} = (\cos 2\theta)(2) - 1 \qquad \frac{dy}{d\theta} = (-\sin 2\theta)(2) + 2(\cos\theta)(1)$$

$$\frac{dx}{d\theta} = 2\cos 2\theta - 1 \qquad \frac{dy}{d\theta} = 2\cos\theta - 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\theta - 2\sin 2\theta}{2\cos 2\theta - 1}$$

TRIG

$$= \frac{2\cos\theta - 2(2\sin\theta\cos\theta)}{2(1 - 2\sin^2\theta) - 1}$$

$$= \frac{2\cos\theta - 4\sin\theta\cos\theta}{2 - 4\sin^2\theta - 1}$$

$$= \frac{2\cos\theta(1 - 2\sin\theta)}{1 - 4\sin^2\theta} \longrightarrow (1)^2 - (2\sin\theta)^2$$

$$(1 + 2\sin\theta)(1 - 2\sin\theta)$$

$$= \frac{2\cos\theta\,\cancel{(1 - 2\sin\theta)}}{(1 + 2\sin\theta)\cancel{(1 - 2\sin\theta)}}$$

$$= \frac{2\cos\theta}{1 + 2\sin\theta}$$

9 The parametric equations of a curve are

$$x = \frac{4t}{2t+3}, \quad y = 2\ln(2t+3).$$

(i) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

$\frac{dy}{dx}$

9709/33/O/N/12

$$x = \frac{4t}{2t+3}$$

$$\frac{dx}{dt} = \frac{(2t+3)(4) - (4t)(2)}{(2t+3)^2}$$

$$\frac{dx}{dt} = \frac{8t + 12 - 8t}{(2t+3)^2}$$

$$\frac{dx}{dt} = \frac{12}{(2t+3)^2}$$

$$y = 2\ln(2t+3)$$

$$\frac{dy}{dt} = 2\,\frac{1}{2t+3} \times 2$$

$$\frac{dy}{dt} = \frac{4}{2t+3}$$

$$\frac{dy}{dx} = \frac{4}{2t+3} \div \frac{12}{(2t+3)^2}$$

$$= \frac{\cancel{4}\,1}{\cancel{2t+3}} \times \frac{(2t+3)^{\cancel{2}}}{\cancel{12}\,3}$$

$$\boxed{\frac{dy}{dx} = \frac{2t+3}{3}}$$

put $x = 1$???

(ii) put $x = 1$ and find t

$$x = \frac{4t}{2t+3}$$

$$1 = \frac{4t}{}$$

$2t+3$

$2t+3 = 4t$

$3 = 2t$

$t = 1.5$

$\frac{dy}{dx} = \frac{2t+3}{3} = \frac{2(1.5)+3}{3} = \boxed{2}$

9709/03/O/N/08

3 The parametric equations of a curve are

$$x = a\cos^3 t, \quad y = a\sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin t + y\cos t = a\sin t\cos t.$$ [3]

(iii) Hence show that, if this tangent meets the x-axis at X and the y-axis at Y, then the length of XY is always equal to a. [2]

9709/03/M/J/09

(i) $x = a\cos^3 t$

$x = a(\boxed{\cos t})^3$

$\frac{dx}{dt} = a(3)(\cos t)^2(-\sin t \times 1)$

$\frac{dx}{dt} = -3a\sin t\cos^2 t$

$y = a\sin^3 t$

$y = a(\boxed{\sin t})^3$

$\frac{dy}{dt} = a(3)(\sin t)^2(\cos t \times 1)$

$\frac{dy}{dt} = 3a\sin^2 t\cos t$

$\frac{dy}{dx} = \frac{\cancel{3a}\sin^{\cancel{2}} t\,\cancel{\cos t}}{-\cancel{3a}\,\cancel{\sin t}\,\cos^{\cancel{2}} t} = -\frac{\sin t}{\cos t} = -\tan t.$

(ii) Tangent: $\frac{dy}{dx} = -\tan t = -\frac{\sin t}{\cos t}$, $x = a\cos^3 t$

$y = a\sin^3 t$

$$y - a\sin^3 t = -\frac{\sin t}{\cos t}(x - a\cos^3 t)$$

$$y\cos t - a\sin^3 t\cos t = -x\sin t + a\sin t\cos^3 t$$

$$x\sin t + y\cos t = a\sin^3 t\cos t + a\sin t\cos^3 t$$

$$x\sin t + y\cos t = a\sin t\cos t(\sin^2 t + \cos^2 t)$$

$$x\sin t + y\cos t = a\sin t\cos t(1)$$

$$x\sin t + y\cos t = a\sin t\cos t$$

(iii) Tangent: $x\sin t + y\cos t = a\sin t\cos t$

x-axis at X	y-axis at Y
$y = 0$	$x = 0$
$x\sin t + 0\cos t = a\sin t\cos t$	$0\sin t + y\cos t = a\sin t\cos t$
$x\sin t = a\sin t\cos t$	$y\cos t = a\sin t\cos t$
$x = a\cos t$	$y = a\sin t$
$X(a\cos t, 0)$	$Y(0, a\sin t)$

$$XY = \sqrt{(a\sin t - 0)^2 + (0 - a\cos t)^2}$$

$$= \sqrt{a^2\sin^2 t + a^2\cos^2 t}$$

$$= \sqrt{a^2(\sin^2 t + \cos^2 t)}$$

$$= \sqrt{a^2(1)}$$

$= \sqrt{a^2}$

$xy = a$

TYPE 3 : IMPLICIT DIFFERENTIATION

y will not be subject of our equation

Do not even try to make y subject.

RULE 1 : $x \xrightarrow{\text{diff}} 1$ $\qquad \frac{d\boxed{}}{dx} \qquad \frac{d\boxed{x}}{dx} = 1$

RULE 2 : $y \longrightarrow \frac{dy}{dx}$ $\qquad \frac{d\boxed{y}}{dx}$

RULE 3 : STRICT DIFFERENTIATION WITH PROPER ATTENTION TO DIFF OF BOX.

Q: Find $\frac{dy}{dx}$: $\quad x^3 + 4x^2y + y^3 = 10$

$$x^3 + 4\underbrace{x^2}_{u}\underbrace{y}_{v} + y^3 = 10$$

Diff

$$3(x)^2(1) + 4\left[x^2\left(\frac{dy}{dx}\right) + y\left[2x^1(1)\right]\right] + 3(y)^2\left(\frac{dy}{dx}\right) = 0$$

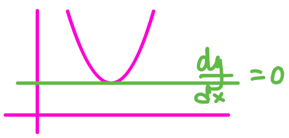
$$3x^2 + 4x^2\frac{dy}{dx} + 8xy + 3y^2\frac{dy}{dx} = 0$$

$$4x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -8xy - 3x^2$$

$$(4x^2+3y^2)\frac{dy}{dx} = -8xy-3x^2$$

$$\frac{dy}{dx} = \frac{-8xy-3x^2}{4x^2+3y^2}$$

2 The equation of a curve is $x^3 + 2y^3 = 3xy$.



(i) Show that $\dfrac{dy}{dx} = \dfrac{y-x^2}{2y^2-x}$. [4]

$x \neq 0, y \neq 0$

(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

grad of tangent $= 0$

$\frac{dy}{dx} = 0$

$$x^3 + 2y^3 = 3xy$$

$$3x^2(1) + 6y^2\frac{dy}{dx} = 3\left[x\frac{dy}{dx} + y(1)\right]$$

$$3x^2 + 6y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y$$

$$6y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 3x^2$$

$$(6y^2-3x)\frac{dy}{dx} = 3y-3x^2$$

$$\frac{dy}{dx} = \frac{3y-3x^2}{6y^2-3x}$$

$$\frac{dy}{dx} = \frac{\cancel{3}(y-x^2)}{\cancel{3}(2y^2-x)}$$

$$\frac{dy}{dx} = \frac{y-x^2}{2y^2-x}$$

(ii) $\frac{dy}{dx} = 0$

$$\frac{y-x^2}{2y^2-x} = 0$$

$$y - x^2 = 0$$

$$y = x^2$$

IN IMPLICIT WHEN WE PUT $\frac{dy}{dx} = 0$, we do not get a value for x. We get another equation.

Solve this simultaneously with original equation.

$$y = x^2 \qquad x^3 + 2y^3 = 3xy$$

$$x^3 + 2(x^2)^3 = 3x(x^2)$$

$$x^3 + 2x^6 = 3x^3$$

$$2x^6 = 3x^3 - x^3$$

$$\cancel{2}x^{\cancel{6}\,3} = \cancel{2}x^{\cancel{3}}$$

$$x^3 = 1$$

$$x = 1$$

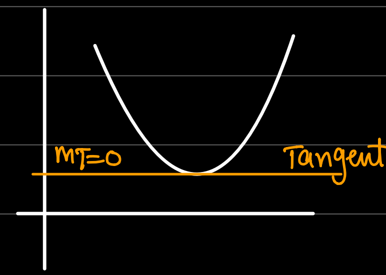
$$y = (1)^2 = 1$$

$$(1, 1)$$

$\boxed{\text{IMP}}$

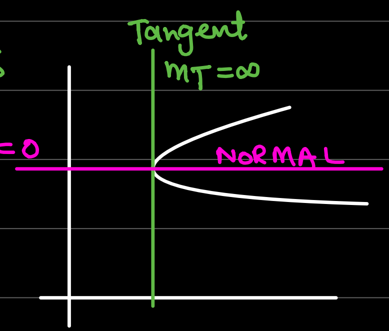
1) TANGENT PARALLEL TO X-AXIS

$$\frac{dy}{dx} = 0$$



2) TANGENT is PARALLEL TO y-AXIS

$\frac{dy}{dx} = \infty$ (you cannot write this in an equation)



$\frac{dy}{dx} = m_T \xrightarrow[\text{reciprocal.}]{\text{Negative}} m_N$

Now put $m_N = 0$

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

$m_T \longrightarrow m_N = 0$

 9709/31/O/N/12

$$\ln(xy) - y^3 = 1$$

$$\frac{1}{xy}\left[x\frac{dy}{dx} + y(1)\right] - 3y^2\left(\frac{dy}{dx}\right) = 0$$

$$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x} - 3y^2\frac{dy}{dx} = 0$$

$$\frac{1}{x} = 3y^2\frac{dy}{dx} - \frac{1}{y}\frac{dy}{dx}$$

$$\frac{1}{x} = \frac{dy}{dx}\left(3y^2 - \frac{1}{y}\right)$$

$$\frac{1}{x} = \frac{dy}{dx}\left(\frac{3y^3-1}{y}\right)$$

$$\frac{y}{x} = \frac{dy}{dx}(3y^3-1)$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x(3y^3-1)}}$$

(ii) $m_T = \frac{dy}{dx} = \frac{y}{x(3y^3-1)}$

$m_N = -\frac{x(3y^3-1)}{y}$ $\qquad$ $m_N = 0$

$$-\frac{x(3y^3-1)}{y} = 0$$

$$-x(3y^3-1) = 0$$

$-x = 0$ $\qquad$ $3y^3-1=0$

$x = 0$ $\qquad$ $y^3 = \frac{1}{3}$

rejected.

$$y = \sqrt[3]{\frac{1}{3}} = 0.693$$

$$\ln(xy) - y^3 = 1$$

$$\ln(0.693x) - 0.693^3 = 1$$

$$\ln(0.693x) = 1.3328$$

$0.693x = e^{1.3328}$

$x =$

Aaj 4–5 ?? <